

# Where is a Marginally Stable Last Circular Orbit in Super-Critical Accretion Flow?

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(Received 2003 June 3; accepted 2003 August 20)

## Abstract

Impressed by the widespread misunderstanding of the issue, we return to the old question of the location of the inner edge of accretion disk around black hole. We recall the fundamental results obtained in the 1970's and 1980's by Warsaw and Kyoto research groups that proved, in particular, that the inner edge does not coincide with the location of the innermost stable Keplerian circular orbit. We give some novel illustrations of this particular point and of some other fundamental results obtained by Warsaw and Kyoto groups. To investigate the flow dynamics of the inner edge of accretion disk, we carefully solve the structure of the transonic flow and plot the effective potential profile based on the angular-momentum distribution calculated numerically. We show that the flow does not have a potential minimum for accretion rates,  $\dot{M} \gtrsim 10L_E/c^2$  (with  $L_E$  being the Eddington luminosity and  $c$  being the speed of light). This property is realized even in relatively small viscosity parameters (i.e.,  $\alpha \sim 0.01$ ), because of the effect of pressure gradient. In conclusion, the argument based on the last circular orbit of a test particle cannot give a correct inner boundary of the super-critical flow and the inner edge should be determined in connection with radiation efficiency. The same argument can apply to optically thin ADAF. The interpretation of the observed QPO frequencies should be re-considered, since the assumption of Kepler rotation velocity can grossly over- or underestimate the disk rotation velocity, depending on the magnitude of viscosity.

**Key words:** accretion: accretion disks, black holes — stars: X-rays

## 1. Introduction

For the reason already explained in the Abstract, we return to the old question where the inner edge  $r_{\text{in}}$  of black hole accretion disk is located. This, and other questions concerning the transition region where the flow of accreted matter changes its character from being almost circular to being almost free-fall, have been intensely studied, and most of them definitely answered, in the late seventies and early eighties by two research groups: the Warsaw group lead by Bohdan Paczyński, and by the Kyoto group lead by Shoji Kato.

Researchers in Warsaw and Kyoto formulated several exact, simple, and practically useful analytic theorems and formulae, that were very much independent of the nature of dissipation, radiative processes and viscosity. They gave a general description of the innermost part of black hole accretion flow, i.e. the region located between the black hole horizon  $r_g$  and the marginally stable circular Keplerian orbit  $r_{\text{ms}}$ . Later developments in hydrodynamics and magnetohydrodynamics of accretion disks have brilliantly confirmed these results. This includes detailed numerical models of supercritical (accretion rate higher than the Eddington one), optically thick and geometrically either thick or slim accretion disks, and sub-critical, optically thin and geometrically thick ADAFs and iron tori (see Kato et al. 1998 for a review of these models).

Questions concerning the transition region and the inner edge of the disk cannot be properly addressed in the

“standard” Shakura – Sunyaev thin accretion disk model because of two crucial mathematical simplifications that the standard model makes — that the angular momentum of the accreted matter  $\ell(r)$  is everywhere Keplerian,  $\ell(r) = \ell_K(r)$ , and that the viscous heating is locally balanced by radiative cooling. In outer parts of the disk,  $r \gg r_{\text{ms}}$ , these assumptions are quite acceptable in the sense that several calculated disk's properties, in particular the spectra, do not depend much on them. They fail completely in the innermost part of the flow,  $r < r_{\text{ms}}$ , where they give wrong qualitative picture of the flow.

Abramowicz and Kato (1989) gave a useful, short and adequate summary of the early Warsaw and Kyoto research with explicit quotations to authors of particular results. Here in this Introduction we repeat three points of their summary in a different order and wording, and without references. All the statements below are general, proven, theorems confirmed later by 3D numerical super-computer simulations (see Igumeshchev et al. 2003 for references to papers describing results of these simulations):

[1] Dynamics. In the innermost part of the disk angular momentum of accreted matter is not Keplerian. The dynamics of the flow depends mostly on the magnitude of the angular momentum there: flows with low angular momenta have very different properties from those with high angular momenta.

The low angular momentum, Bondi-type flows have  $\ell(r) < \ell_{\text{ms}} = \ell_K(r_{\text{ms}})$ . The centrifugal force is dynamically unimportant everywhere, and the flow is far from mechan-

ical equilibrium. Flow lines are never close to circles, and the flow properties are similar to these known from the case of a spherical accretion. For the Bondi-type flows, the very concept of the inner edge loses its usefulness: in the Bondi-type of accretion flows there is simply no characteristic radius that could be called the inner edge of the flow.

The high angular momentum, disk-type accretion flows have,  $\ell(r) < \ell_{\text{ms}}$ . The outer part of the flow, i.e. the accretion disk itself, is close to mechanical equilibrium determined by gravitational, centrifugal, and pressure forces. The total gravitational plus centrifugal potential  $\psi_{\text{eff}}(r, z)$ , has two extrema at locations  $r_*$  given as the two solutions of the equation  $\ell(r_*) = \ell_K(r_*)$ , i.e. where the angular momentum of the matter  $\ell(r_*)$  equals to the corresponding Keplerian value at the same radius  $r_*$ . Note, that because the relativistic Keplerian angular momentum distribution has a minimum at  $r_{\text{ms}}$  and we are now interested in the case  $\ell(r) > \ell_{\text{ms}}$ , there must be two such locations on both sides of  $r_{\text{ms}}$ . The first one,  $r_* = r_{\text{cent}} > r_{\text{ms}}$ , corresponds to a minimum of  $\psi_{\text{eff}}(r, z)$  and a maximum of the pressure. The second one,  $r_* = r_{\text{cusp}} < r_{\text{ms}}$  corresponds to a maximum of  $\psi_{\text{eff}}(r, 0)$  at the equatorial plane  $z = 0$ . The equipotential surface  $\psi_{\text{eff}}(r, z) = \text{const} = \psi_{\text{eff}}(r_{\text{cusp}}, 0)$  crosses itself along the circle  $(r, z) = (r_{\text{cusp}}, 0)$ . This self-crossing equipotential is called the Roche lobe. Inside the disk, i.e. for  $r > r_{\text{cusp}}$ , matter very slowly moves inward due to the action of viscous torques. For  $r < r_{\text{cusp}}$  it goes inward very fast, indeed transsonically, due to the insufficient centrifugal support and therefore lack of mechanical equilibrium. This is similar to the more familiar situation of the Roche lobe overflow known in the context of close binaries.

Thus, in the region close to the location of the cusp,  $r = r_{\text{cusp}}$ , the flow changes its character from almost circular to almost free-fall, and for this reason, the cusp represents exactly what it is the crucial physical meaning of the “inner edge” of the disk. The other possibility would be the sonic radius  $r_{\text{sonic}}$ , where the inward radial velocity of the flow changes from sub- to super-sonic. The sonic radius is just slightly closer to the black hole than the cusp,  $r_{\text{mb}} < r_{\text{sonic}} < r_{\text{cusp}} < r_{\text{ms}}$ .

[2] Viscosity and two types of the flow. For stationary accretion flow that asymptotically, i.e. for large radii, are described by the standard Shakura – Sunyaev model, and have viscosity given by the standard  $\alpha$  prescription, the value of the angular momentum in the inner part of the disk, expressed in the dimensionless units of  $\ell_K(r_{\text{ms}})$ , depends on  $\alpha$  and accretion rate  $\dot{M}$  (Muchotrzeb and Paczyński 1982; Matsumoto et al. 1984).

For any value of  $\dot{M}$  there is a critical value of  $\alpha_{\text{crit}}$  such that for  $\alpha > \alpha_{\text{crit}}$  the flow is of the Bondi type, while for  $\alpha < \alpha_{\text{crit}}$  the flow is of the disk type. For small very subcritical accretion rates  $\alpha_{\text{crit}} < 0.1$

[3] Location of the inner edge and efficiency. Assuming “no torque inner boundary conditions”, i.e. that viscous torque cannot act across the black hole horizon, location of the cusp is directly connected to the efficiency of accretion.

The efficiency equals to minus the binding orbital energy at the inner edge divided by speed of light square,  $\eta = -e_K(r_{\text{in}})/c^2$ . In particular, for accretion disks that are radiatively inefficient  $\eta \approx 0$ , i.e. thick disks, slim disks and ADAFs, the inner edge approaches the location of the marginally bound circular Keplerian orbit  $r_{\text{mb}}$ , because  $\eta(r_{\text{mb}}) = 0$ . For the Schwarzschild i.e. non-rotating black hole with the mass  $M$ , it is  $r_g = 2GM/c^2$ ,  $r_{\text{mb}} = 2r_g$ , and  $r_{\text{ms}} = 3r_g$ .

These fundamental results of Warsaw and Kyoto groups have been reviewed many times, also in the best known textbooks and monographs on accretion theory (Frank, King and Raine 2002 3rd edition; Kato, Fukue and Mineshige 1998; Abramowicz, Björnsson and Pringle 1998). Some of them belong to the most often quoted results in the black hole accretion disk theory. It is therefore quite surprising, that still today there is a widespread and wrong conviction that the innermost edge of the accretion disk must coincide with the location of the marginally stable circular Keplerian orbit  $r_{\text{ms}}$ . This wrong opinion appears to be especially popular among observers. While it is quite possible (as often suggested) that the highest frequency of quasi periodic variability observed in many of these sources may come from blobs of matter orbiting the central black hole or neutron star close to the inner edge of the disk, one cannot claim that the observed highest frequency equals the orbital frequency at the marginally stable circular Keplerian orbit.

This article is another attempt (see Paczyński, 1998) to clear the issue of the inner edge of the disk by reminding the classic early results of Warsaw and Kyoto groups. Here we are discussing some new illustrations of them. We do not always quote names of the original Warsaw and Kyoto authors of particular classical results.

In this paper, we thus discuss the stability of circular orbits. In section 2, we explicitly show the shape of the effective potential and examined the stability. In section 3, we discuss the unique properties of the transonic regions of the slim disk and their observational implications. The last section is devoted to conclusions.

## 2. Effective Potentials of Accretion Disk

### 2.1. Basic Considerations

It has been shown by the X-ray observations of black hole candidates (BHCs) that the radius of the inner boundary estimated through the spectral fitting to the X-ray data is constant in time despite large-amplitude variations in luminosity, and it coincides with  $3r_g$  within error bars (e.g. Makishima et al. 1986; Tanaka & Shibazaki 1996; Ebisawa 1999).

Caution should be taken regarding this argument, however, since all the accreting material should eventually pass through the unstable regions before finally falling across the event horizon. That is, certain amount of materials should at any times exist in the unstable regions. (This contrasts the case of a static system, in which we may argue that there are practically no materials present in unstable regions.) Although matter density is non-zero

inside  $3 r_g$ , the unstable region has no observational significance for BHCs with luminosity being well below the Eddington luminosity, since these regions are optically thin, thus emitting fewer photons. Some observational effect appears when the luminosity becomes comparable to the Eddington luminosity, for which the flow inside  $3 r_g$  might become optically thick, thus producing significant soft X-ray emission (Watarai et al. 2000). Some ULXs (ultra-luminous X-ray sources) show a trend that the apparent inner-edge of the disk decreases with increase of luminosity (Mizuno, Kubota, & Makishima 2001; Watarai, Mizuno, & Mineshige 2001). Microquasar GRS1915+105 exhibits rapid changes of the inner edge radius during transitions (Belloni et al. 1997; Yamaoka et al. 2001; Watarai & Mineshige 2003).

It has been claimed that usual arguments regarding the inner boundary of the disk based on the particle orbits are sometime misleading in the transonic flow (see Kato, Fukue, & Mineshige 1998 Chapter 2 for a review). To see this problem more clearly, let us first recall the basic argument on the stability of the circular orbits around a non-rotating black hole. General relativistic description of the effective potential leads

$$\psi_{\text{eff}}^{GR}(r) = -\frac{GM}{r} \left( 1 + \frac{\ell^2}{c^2 r^2} \right) + \frac{\ell^2}{2r^2} \quad (1)$$

(see, e.g., Shapiro & Teukolsky 1983, sec 12.4). Here,  $\ell (\equiv r v_\varphi)$  denotes the specific angular momentum of the test particle and the last term on the right-hand side corresponds to the potential of the centrifugal force. The general relativistic effect appears in the term in proportion to  $r^{-3}$ . It is this term that makes mass accretion to a black hole possible, even when matter has a finite angular momentum. A particle having angular momentum ( $\ell$ ) greater than the critical value,  $\ell_{\text{crit}}$  (which is on the order of  $c r_g$ ), can rotate on a stable circular orbit at a certain radius determined by the minimum of the effective potential of constant  $\ell$ . As  $\ell$  decreases, this equilibrium radius decreases and, when  $\ell < \ell_{\text{crit}}$ , it disappears. Thus, the minimum value of the equilibrium radius is  $3 r_g$  for a non-rotating black hole. This argument is directly applicable to the case of general relativistic fluid (Lu et al. 1995), and the marginally stable orbit in such a case is, again,  $\approx 3 r_g$ .

In short, the stability argument assumes the presence of an equilibrium point, in which all the forces are balanced, for a given angular momentum. A problem arises, when the inertial term  $[\propto v_r (dv_r/dr)]$  becomes substantial. This indeed occurs around the transonic point, in which matter velocity equals to sound velocity. Since the flow speed is  $\sim c$  while the sound velocity is at most  $\sim c/\sqrt{3}$  near the event horizon, the flow should be supersonic at small radii but must be subsonic at large radii; that is, there should be a transonic point near the black hole (see Chap. 9 of Kato, Fukue, & Mineshige 1998 for more details).

Again, the problem is not serious in standard-type disks with luminosity,  $L < L_E$ , in which the inner edge has been shown to be barely inside the marginally stable orbit at  $r_{\text{ms}}$  (Muchotrzeb & Paczyński 1982; Muchotrzeb

1983; Matsumoto et al. 1984; Abramowicz & Kato 1989). A noteworthy effect appears in optically thin ADAF (advection-dominated accretion flow) and in near-critical accretion flow (noted as a slim disk). Although transonic nature of the flow has been extensively discussed mainly in the former case (Narayan, Honma, & Kato 1997) and in the context of geometrically thick torus, the place of the inner edge of the flow has been poorly investigated so far in relation to the stability of circular orbits.

## 2.2. Basic equations and numerical procedures

The numerical procedures are the same as those adopted by Watarai et al. (2000, see Chap. 10 of Kato, Fukue, & Mineshige 1998 for the derivations of the basic equations and the detailed discussion). The essential ingredients are the transonic condition of the flow and the presence of advective energy transport. In the present study we adopt the pseudo-Newtonian potential (Paczynski & Wiita 1980) to simplify the treatment. It is known that this potential is a good approximation of the general relativistic one down to  $\sim 2 r_g$  and gives the correct radius of the marginally stable orbit (see section 2.2). Thus, the adoption of the pseudo-Newtonian potential will not introduce serious errors when we are concerned with the properties of the flow at  $r \gtrsim 3 r_g$ .

The basic equations are as follows: The mass conservation reads

$$\dot{M} = -2\pi r \Sigma v_r, \quad (2)$$

where  $v_r$  and  $\Sigma$  are radial velocity and surface density, respectively. The momentum equation in the radial direction is

$$v_r \frac{dv_r}{dr} + \frac{1}{\Sigma} \frac{dW}{dr} = \frac{\ell^2 - \ell_K^2}{r^3} - \frac{W}{\Sigma} \frac{d \ln \Omega_K}{dr}, \quad (3)$$

where  $W \equiv \int p_{\text{tot}} dz$  is the integrated pressure and the Keplerian angular momentum is  $\ell_K = r^2 \Omega_K$ , with  $\Omega_K$  being the Keplerian angular frequency under the pseudo-Newtonian potential. Note that the last term on the right-hand side is a correction term for gravity.

The angular momentum conservation is written as

$$\dot{M}(\ell - \ell_{\text{in}}) = -2\pi r^2 T_{r\varphi}, \quad (4)$$

where  $T_{r\varphi}$  is the height integrated viscosity stress tensor and we adopt the  $\alpha$  viscosity prescription;

$$T_{r\varphi} \equiv -\alpha p_{\text{tot}} H. \quad (5)$$

Here,  $H$  denotes the half-thickness of the disk and is

$$H = (p_{\text{tot}}/\rho\Omega)^{1/2} \quad (6)$$

from the hydrostatic balances in the vertical direction. The equation of state is

$$p_{\text{tot}} = p_{\text{gas}} + p_{\text{rad}} = \frac{\Re}{\bar{\mu}} \rho T + \frac{a}{3} T^4, \quad (7)$$

where  $\Re$ ,  $\bar{\mu}$  and  $a$  are the gas constant, mean molecular weight, and the radiation constant, respectively. Note that all the quantities refer to the values on the equatorial plane.

The energy equation involves viscous heating, radiative cooling, and advective cooling;

$$Q_{\text{vis}}^+ = Q_{\text{rad}}^- + Q_{\text{adv}}^- \quad (8)$$

where each term is explicitly written as

$$Q_{\text{vis}}^+ = r T_{r\varphi} \frac{d\Omega}{dr} \quad (9)$$

$$Q_{\text{rad}}^- = \frac{8acT^4}{3\bar{\kappa}\rho H}, \quad (10)$$

$$Q_{\text{adv}}^- = \frac{9}{8} v_r \Sigma T \frac{ds}{dr}. \quad (11)$$

Here,  $s$  denotes specific entropy in the equatorial plane and  $\bar{\kappa}$  is the Rosseland-mean opacity.

We solved the set of equations (2)–(8) from the outer boundary at  $10^4 r_g$  inward down to  $\sim r_g$  through the transonic point. The inner boundary is free.

### 2.3. Effective potentials of high $\alpha$ disks

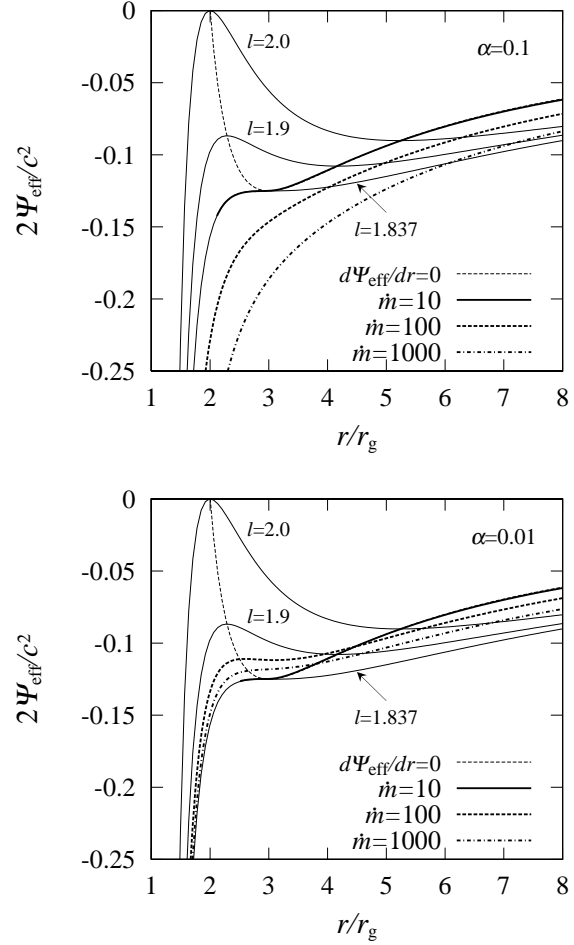
The expression for the effective potential is then

$$\psi_{\text{eff}}^{\text{PN0}}(r) = -\frac{GM}{r - r_g} + \frac{\ell^2}{2r^2}. \quad (12)$$

If we fixed  $\ell$ , the condition of  $d\psi_{\text{eff}}(r)/dr = 0$  usually gives three solutions, among which only one solution is stable, since it corresponds to a potential minimum,  $d^2\psi_{\text{eff}}(r)/dr^2 > 0$ , and the other two are unstable solutions with  $d^2\psi_{\text{eff}}(r)/dr^2 < 0$ . We vary  $\ell$  and seek for a solution which satisfies the condition,  $d^2\psi_{\text{eff}}(r)/dr^2 = 0$ , at the radius where  $d\psi_{\text{eff}}(r)/dr = 0$  holds. We find the solution for  $\ell_{\text{crit}} = 1.837$  and the derived radius of the marginal stable orbit is  $r_{\text{ms}} = 3 r_g$ , just the same as that derived based on the Schwarzschild metric. When the angular momentum of a test particle is less than this critical value, there no longer exists a stable circular orbit.

Next, we consider the case of fluid and calculate the effective potential based on the numerical models. We adopt the numerical value of specific angular momentum, which is a function of radius, to calculate the effective potential. Figure 1 shows the radial profile of the effective potential,  $\psi_{\text{eff}}^{\text{PN0}}$ , for various accretion rates,  $\dot{m} = 10, 100$ , and  $1000$ . Here, we define the normalized accretion rate to be  $\dot{m} \equiv \dot{M}c^2/L_E$ . We fix the black hole mass to be  $M = 10 M_\odot$  in the entire calculations. For comparison, we also plot the effective potentials of fixed angular momenta,  $\ell = 2.0, 1.9$  and  $1.837 (= \ell_{\text{crit}})$ . The thin dotted line represents the locus of the stable orbits; namely, we connect the points where  $d\psi_{\text{eff}}/dr = 0$  and  $d^2\psi_{\text{eff}}(r)/dr^2 > 0$  hold for various values of  $\ell$  ( $\geq \ell_{\text{crit}}$ ). This is the trajectory along which accreting gas would move if the gas had the Keplerian rotation velocity (i.e., if the inertial and pressure terms were zero; see equation [3]).

Let us compare the numerical solutions with the analytical ones (fixed  $\ell$  solutions) for the cases with relatively high  $\alpha = 0.1$  (see the upper panel). We notice that both well agree for small accretion rates with  $\dot{m} = 1$  and  $10$ . This result justifies the usual belief that the inner edge of the disk is determined by the marginal stable orbit. The



**Fig. 1.** Profiles of the effective potentials for  $\alpha = 0.1$  (upper panel) and  $\alpha = 0.01$  (lower panel), respectively. The thick solid, dashed, dotted lines represent the numerically calculated potential for accretion rates of  $\dot{m} (\equiv \dot{M}c^2/L_E) = 10, 100$ , and  $1000$  from the top to bottom. The thin solid lines show the same but for a fixed specific angular momentum of  $\ell (\equiv r v_\varphi) = 2.0, 1.9$ , and  $1.837$  from the top to bottom. The thin dotted line represents  $\psi_{\text{eff}} = 0$  curve.

sonic points lie slightly inside  $3 r_g$ , as was already pointed out (Matsumoto et al. 1984). For large accretion rates,  $\dot{m} \geq 100$ , in contrast, big discrepancies appear. The rotation velocity is significantly lower than the Keplerian value, which causes the downward shift of the potential profile. The larger  $\dot{m}$  is, the lower becomes the value of the potential, indicating a higher efficiency of angular-momentum removal. Due to this reason, such an accretion flow is called as viscosity-driven flow. As a consequence, we see  $\ell < \ell_{\text{crit}}$  at  $r \lesssim 4 r_g$  for  $\dot{m} = 10$  and at  $r \lesssim 6 r_g$  for  $\dot{m} = 100$ ; i.e., always gravity dominates over the centrifugal force and the inertial term gives an important contribution in the momentum equation. Since there is no minimum in their effective potential profile, there is no equilibrium place and, hence, the usual technique of the stability analysis cannot be applied here. Note that our derived angular-momentum distribution agrees well with those found by Abramowicz et al. (1988) and Artemova



et al. (2001), but they did not show the effective potential profiles.

#### 2.4. Effective potentials of low $\alpha$ disks

Let us turn to the cases of small  $\alpha = 0.01$  (see the lower panel of figure 1). The behavior of the low  $\dot{m}$  disk does not differ from the previous cases, but that of high  $\dot{m}$  disk is qualitatively different; most notably, the angular momentum at  $3 r_g$  still exceeds the critical value. This means that the rotation velocity is super-Keplerian. Nevertheless, accretion is possible because of enhanced pressure gradient force. In fact, we find a pressure peak at  $4 \sim 5 r_g$  and, hence, the pressure gradient force can induce gas accretion flow at  $r \lesssim 4 r_g$ . Hence, this kind of flow is called as pressure-driven flow. The potential profile has a minimum, even when  $\dot{m} \geq 100$ , which makes a good contrast with the low  $\alpha$  disks. The radius of the potential minimum is around  $3 r_g$ . However, we cannot simply conclude that an orbit at this radius is stable, since matter cannot stay at this radius. In other words, this is not an equilibrium radius.

#### 2.5. Effect of pressure gradient

We have basically confirmed results of previous authors at low accretion rates <sup>1</sup>. In addition, we find that accretion rates are another key parameter to determine the types of the accretion processes. Note, however, that equation (12) does not include the effect of pressure gradient. Hence, we modify the effective potential as,

$$\psi_{\text{eff}}^{\text{PN1}}(r) = -\frac{GM}{r-r_g} + \frac{\ell^2}{2r^2} - \int_r^{r_{\text{out}}} \frac{1}{\Sigma} \left( \frac{dW}{dr} \right) dr. \quad (13)$$

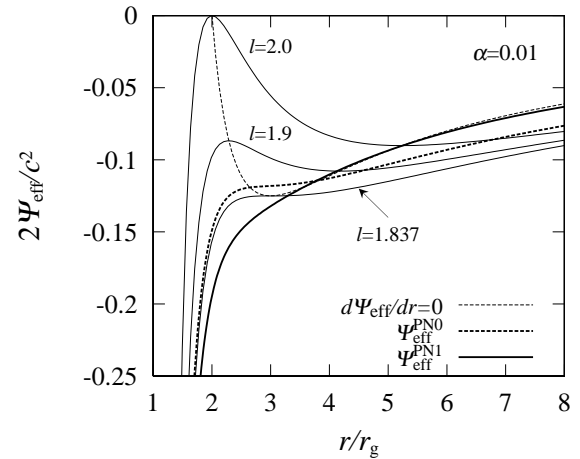
Here, the third term describe the change of the internal energy by work done by the pressure gradient <sup>2</sup>. This term is important to understand the behavior of a fluid element because the flow becomes radiation pressure dominated under high mass accretion rate.

Figure 2 shows the contributions of pressure gradient term for  $\dot{m} = 1000$  case. The thick dotted lines on figure 2 are calculated by equation (13); i.e., the effect of pressure gradient is excluded. In such cases, we can distinguish the flow type with the  $\alpha$  parameters. For large  $\alpha$  (upper panel in figure 2), the flow is driven by viscosity then there is no potential minimum. On the other hand, for smaller  $\alpha$  (lower panel in figure 2), the accretion flow inside  $\sim 3 r_g$  is driven by pressure so that there is a potential minimum even if relatively high mass accretion rate. In both cases, however, if we include the pressure term (i.e., the thick solid lines, which are derived from equation (13)), then the effective potential does not have minimum. Pressure gradient term is not so important for large  $\alpha$  case, because the flow is basically viscosity-driven. However, for small  $\alpha$  case, the form of the effective potential significantly changed due to the effect of pressure gradient. To induce accretion onto black hole for small  $\alpha$ , accretion must be

driven by the pressure gradient force, and the disk must have a pressure peak in the inner region. Inside of the pressure peak, the pressure gradient force works inward. Conversely, the pressure gradient force works outward outside the pressure peak. Hence, the pressure minimum does not appear even for small  $\alpha$  model <sup>3</sup>. Again, we note that the classical analysis based on equation (1) or (12) is no longer valid in high mass-accretion systems because of the significant effect of radiation pressure force.

#### 2.6. Viscosity driven flow and pressure driven flow

Transonic nature of optically thick accretion disk has been extensively investigated since the early 1980's. Abramowicz et al. (1978) found the cusp structure located between  $r_{\text{ms}}$  and marginally bound orbit,  $r_{\text{mb}}$ , for any angular momentum distribution. They proposed that mass accretion into a black hole may occur in a similar way to the case of the Roche-lobe overflows. Muchotrzeb & Paczyński (1982) were the first to find numerically that the cusp radius is located between  $r_{\text{ms}}$  and  $r_{\text{mb}} (\sim 2 r_g)$ , for a low viscosity parameter,  $\alpha = 0.001$ . Their conclusion is that the “no torque boundary condition at  $r_{\text{in}}$ ” is valid. Matsumoto et al. (1984) have found distinct flow structure in cases with high viscosity parameters and claimed that flow properties differ, depending on the magnitudes of viscosity. For small  $\alpha$  ( $\lesssim 0.05$ ) the pressure peak around  $r \simeq 4 - 5 r_g$  drives the mass accretion, whereas for large  $\alpha$  ( $\gtrsim 0.05$ ) the flow is mainly driven by viscosity.



**Fig. 2.** Profiles of the effective potentials for  $\alpha = 0.01$ . The thick solid lines represent the numerically calculated potential, equation (13), for accretion rates of  $\dot{m} (\equiv Mc^2/L_E) = 1000$ . The thick dotted lines show the calculated potential, but the lines are derived from equation (12).

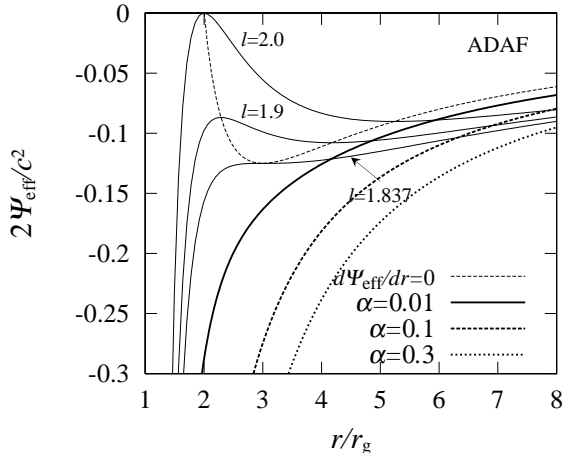
We also calculate optically thin ADAF with different  $\alpha$  parameters and display the resultant effective potential in figure 3. In ADAF, we simply assume one-temperature plasma and bremsstrahlung emission as the cooling process. Other equations are the same as those of the opti-

<sup>1</sup> We also calculated  $\dot{m}=1, 0.1, 0.01$  cases, however, the shape of the effective potential is almost same as  $\dot{m} = 10$ .

<sup>2</sup> We ignored the correction for gravity, since its contribution is small.

<sup>3</sup> We also confirm that there is no radius where  $\psi_{\text{eff}}/dr = 0$  holds, for smaller  $\alpha (=0.001)$  case.

cally thick case. Unlike the case of optically thick disk,



**Fig. 3.** Same as figure 1 but for the optically thin ADAF. The solid lines represent the effective potentials for  $\alpha = 0.01, 0.1, 0.3$  from the top to bottom. We set the accretion rate to be  $\dot{m} = 10^{-3}$ . There is little accretion rate dependence.

the effective potential profile of the optically thin ADAF does not exhibit strong  $\dot{m}$ -dependence. Our result is in good agreement with those by Narayan et al. (1997).

### 3. Discussion

We have demonstrated that a marginally stable orbit does not always exist in high luminosity disks; that is, the inner disk radius is not necessary equal to  $3 r_g$ . This indicates that the inner disk radius derived by the fitting to the observed soft X-ray spectra based on the multi-color disk model (Mitsuda et al. 1984) does not always correspond to  $r_{\text{ms}}$ , especially at high  $\dot{m}$ . The same is true for the ADAF. There should be no problems, however, for low luminosity, standard-type disks with  $L \lesssim L_E$ .

Our results should influence the observational interpretation as to the black hole rotation (Zhang et al. 1997; Makishima et al. 2000). The widely used argument is that if the inner-disk radius estimated by spectral fitting etc. is significantly less than  $3 r_g$ , the black should be a Kerr hole. This argument does not hold for luminous systems shining at  $L \sim L_E$ , since they always show a smaller inner-edge radius (Watarai et al. 2000, 2001).

The same is true for the optically thin ADAF. The asymmetric iron  $K\alpha$  line profiles observed at 6.4 keV in black hole candidates provides a plausible evidence of strong gravitational field of black hole (Fabian et al. 1989; Kojima 1991; Tanaka et al. 1995; Miller et al. 2002), and from the spectral fitting it has been claimed that the central black hole should be rapidly rotating (Iwasawa et al. 1999), since the inner disk radius derived by the fitting is less than  $3 r_g$ . We, however, argue that the inner edge in optically thick accretion disk,  $r_{\text{in}}$ , can be smaller than  $3 r_g$  in the cases with large  $\alpha$  and high  $M$ . Since the effect of gravitational redshift is on the order of  $1/\sqrt{1 - r_g/r}$ , it

might not be so easy to discriminate the effects of black hole rotation and those of super-critical flow solely from the spectral lines and/or the thermal spectral component.

Recently, several groups have succeeded in performing multi-dimensional magneto-hydrodynamic (MHD) simulations of accretion flow and confirmed the presence of a marginally stable orbit around  $3 r_g$  (Hawley & Balbus 1999; Hawley 2000; Armitage et al. 2001; Hawley & Balbus 2002). On the other hand, Krolik & Hawley (2002) proposed the “inner edge” of an accretion disk around a black hole depends on the definition, i.e., turbulence edge, stress edge, reflection edge and radiation edge are defined by each physical process. That is, the often used expression “inner edge” does not always correspond to  $r_{\text{ms}}$ . This paper describes one such example.

### 4. Conclusions

In this paper, we discuss the dynamical properties of the transonic flow in super-critical regimes ( $M \gtrsim 10 L_E/c^2$ ) and demonstrate that the marginally stable orbit does not exit for high mass accretion rates ( $\dot{m} \gtrsim 10$ ). For small viscosity parameters ( $\alpha \lesssim 0.01$ ), the effect of pressure gradient is important for discussion of the marginal stable orbit, since the effective potential including the pressure gradient does not have a minimum. That is, the usual argument based on the assumption that the inner edge of the disk coincides with the marginally stable orbit does not work here. The inner-disk radius obtained by the spectral fitting does not necessarily coincide with the radius of the marginally stable orbit. The observed value is more like the so-called radiation edge, outside which substantial emission is produced. We cannot simply relate the marginal stable orbit with the inner boundary. To determine the radiation edge precisely, we need to solve full radiation transfer equations in more than two dimensions, which is left as future work.

We would like to thank the referee M.A. Abramowicz for his critical comments and suggestions. We are also grateful to S. Kato, W. Kluzniak, J. Fukue, K. Ohsuga, Y. Kato, and R. Takahashi for their useful comments and discussions. This work was supported in part by the Grants-in Aid of the Ministry of Education, Science, Sports, and Culture of Japan (14001680, KW; 13640328, 14079205, SM).

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